Exercise 13

Find the closed form function for the following Taylor series:

$$f(x) = 2 + 2x + \frac{1}{2!}x^2 - \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 + \cdots$$

[TYPO: This should be a plus sign.]

Solution

In order to get the answer at the back of the book, the sign of the x^3 term should be positive.

$$f(x) = 2 + 2x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 + \cdots$$
$$f(x) = 1 + x + 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 + \cdots$$
$$f(x) = 1 + x + \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 + \cdots$$
$$f(x) = 1 + x + \sum_{n=0}^{\infty} \frac{1}{n!}x^n$$

Therefore,

$$f(x) = 1 + x + e^x.$$